The Parameters of Nonlinear Devices from Harmonic Measurements*

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Summary—A method is discussed for determining the parameters of a nonlinear device by measurement of the harmonics generated in the output when a sinusoidal wave is applied at the input. Formulas are developed, in terms of the measured quantities, for determining the coefficients of the power series describing the input-output characteristic and the coefficients of the Fourier series giving the harmonic conversion transconductance.

Introduction

THE technique has been described[1] for utilizing the measured Fourier coefficients of an output waveform resulting from the application of a sinusoidal input wave to a nonlinear device for the purpose of obtaining its power series characteristic. The general relation giving the power series coefficients in terms of the Fourier coefficients has not been given heretofore, to the authors' knowledge, though formulas for a limited number of terms are known.[2] Such a general relation is developed here.

In addition to the foregoing, the quantity referred to here as "harmonic conversion transconductance" can be obtained by similar techniques. Harmonic conversion transconductance was discussed by Herold,[3] who developed a method for determining it by a point-by-point analysis of the transconductance characteristic of the device in question. Here this quantity is found again from the Fourier coefficients of the output waveform when a sinusoidal waveform is applied to the input. While harmonic conversion transconductance is not used often by equipment designers, it has been found useful for estimating spurious response levels in radio receivers.

It is assumed that this technique is to be employed at low frequencies where effects such as are caused by finite transit time are negligible. Furthermore, devices with multivalued transfer characteristics are ruled out.

Power Series From Fourier Series

Consider Fig. 1 which shows a nonlinear transfer characteristic such as might be obtained in a vacuum tube and whose characteristic can be expressed by

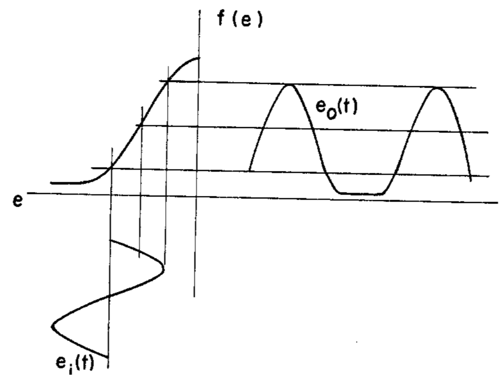

Fig. 1—Nonlinear characteristic and its effect on a sinusoidal waveform.

$$f(e) = \sum_{m=0}^{n} a_m e^m. \tag{1}$$

The output waveform obtained by applying $e = e_1(t) = E \cos t$ will be given by the Fourier series

$$e_0(t) = \frac{b_0}{2} + \sum_{r=1}^{n} b_r \cos rt. \tag{2}$$

The b_r are measurable using a wave analyzer; bear in mind that the polarity of b_r must be determined, too. The latter might be found by comparing the output waveform $e_0(t)$ and the filtered harmonic under consideration on an oscilloscope as indicated in Fig. 2. Assuming no large phase shifts in the analyzer and $e_0(t)$ a maximum at $t = 0$, then a maximum for the harmonic at $t = 0$ (choosing $t = 0$ for even symmetry) indicates a positive term; a minimum indicates a negative term. If $e_0(t)$ is minimum at $t = 0$ the rule is reversed with respect to signs.

The coefficients b_r of (2) are determined also by putting $e = E \cos t$ into (1),

$$e_0(t) = \sum_{m=0}^{n} a_m E^m \cos^m t, \tag{3}$$

and comparing (2) and (3) term by term. The relations between the Fourier coefficients and the power series coefficients then can be obtained.

To effect this comparison, expand $\cos rt$ in (2) as a Tchebycheff polynomial of degree r in powers of $\cos t$.[4,5]

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[1] Staff of the Dept. of Elec. Eng., Mass. Inst. Tech., "Applied Electronics," John Wiley & Sons, Inc., New York, N. Y., p 675; 1943.

[2] An example can be found in F. P. Holder and H. W. Mauldin, Jr., "Study Program for Investigation to Aid in Reduction and Prevention of U.H.F. Interference," Georgia Inst. Tech., Atlanta, Ga., Eng. Experiment Station, Tech. Rep. No. 1, pp. 23–25; March 1, 1955.

[3] E. W. Herold, "The operation of frequency converters and mixers," Proc, IRE, vol. 30, pp. 84–102; February, 1942.

[4] Bateman Manuscript Project, "Higher Transcondental Functions," McGraw-Hill Book Co., Inc., New York, N. Y., p. 185; 1953.

[5] The authors wish to acknowledge the suggestion of the reviewers that use be made of the Tchebycheff polynomial expansion to demonstrate the final result of (6).

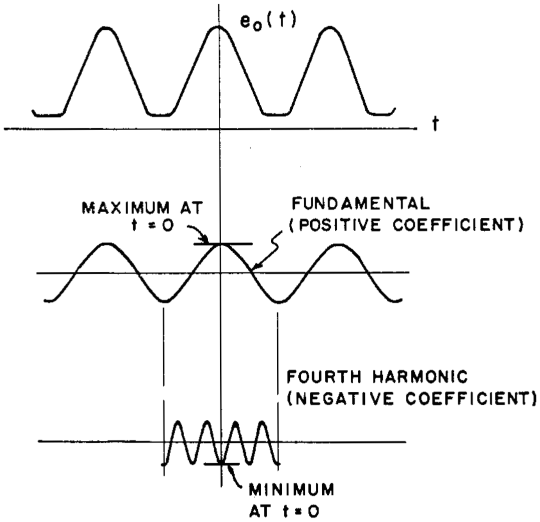

Fig. 2—Decomposition of a complex wave into positive and negative coefficient harmonics.

Thus,

$$\cos rt = T_r(\cos t)$$
$$= \frac{r}{2} \sum_{j=0}^{[r/2]} (-1)^j \frac{(r-j-1)!}{j!(r-2j)!} (2 \cos t)^{(r-2j)}, \quad (4)$$

where the symbol $[r/2]$ denotes the greatest integer not exceeding $r/2$. Putting (4) into (2),

$$e_0(t) = \frac{b_0}{2} + \sum_{r=1}^{n} b_r \frac{r}{2}$$
$$\cdot \sum_{j=0}^{[r/2]} (-1)^j \frac{(r-j-1)!}{j!(r-2j)!} (2 \cos t)^{(r-2j)}. \quad (5)$$

To write this result in the form of (3), note that terms involving $\cos^m t$ in (5) will be obtained whenever $r - 2j = m$. For example, for $j = 0$ and $r = m$ a contribution is obtained,

$$2^{(m-1)} b_m \cos^m t.$$

For $j = 1$ and $r = m + 2$, the component is obtained

$$-2^{(m-1)} b_{(m+2)} (m+2) \cos^m t, \quad \text{etc.}$$

The value of r ranges up to n, or $r_{\max} = n = m + 2j_{\max}$, thus $j_{\max} = (n-m)/2$. In general, the complete coefficient of the term $\cos^m t$ then can be written as

$$a_m E^m = 2^{(m-1)} \left[b_m + \sum_{j=1}^{[(n-m)/2]} (-1)^j \right.$$
$$\left. \cdot (m+2j) \frac{(m+j-1)!}{m!j!} b_{(m+2j)} \right], \quad m = 0, 1, 2, \cdots. \quad (6)$$

Defining $\alpha_m = a_m E^m / 2^{(m-1)}$, the following typical results are had for specific values of m.

$$\alpha_1 = b_1 - 3b_3 + 5b_5 - 7b_7 + \cdots$$
$$\alpha_2 = b_2 - 4b_4 + 9b_6 - 16b_8 + \cdots$$
$$\alpha_3 = b_3 - 5b_5 + 14b_7 - 30b_9 + \cdots$$
$$\alpha_4 = b_4 - 6b_6 + 20b_8 - 50b_{10} + \cdots.$$

Harmonic Conversion Transconductance

The harmonic conversion transconductance associated with the s'th harmonic of an oscillator signal applied to a nonlinear device is defined as one half the coefficient of the s'th term of the series

$$g_m(t) = \sum_{s=0}^{s=n} g_s \cos st \quad (7)$$

where $g_m(t)$ is the time-varying transconductance of the device when an oscillator input wave is applied.

To find the coefficients g_s, the harmonics of the waveform can be measured, as in the preceding section, and related to the coefficients as shown below. To produce a meaningful result, it will be necessary to make the test waveform equal in amplitude to the actual oscillator signal.

If the input waveform $e_1(t)$ is $E \cos t$, the output current will be given by the Fourier series

$$i_0(t) = \sum_{s=0}^{s=n} b_s \cos st. \quad (8)$$

At the same time, the transconductance will be given by

$$g_m(t) = \frac{d[i_0(t)]}{dt} \Big/ \frac{de_1(t)}{dt}. \quad (9)$$

Since

$$\frac{d[i_0(t)]}{dt} = -\sum_{s=0}^{s=n} s b_s \sin st$$

and

$$\frac{d[e_1(t)]}{dt} = -E \sin t$$

then

$$g_m(t) = \frac{1}{E} \sum_{s=1}^{s=n} \frac{s b_s \sin st}{\sin t}$$
$$= \frac{1}{E} \sum_{\mu=0}^{\mu=(n_0-1)/2} \frac{(2\mu+1) b_{(2\mu+1)} \sin (2\mu+1)t}{\sin t}$$
$$+ \frac{1}{E} \sum_{\mu=1}^{\mu=n_e/2} \frac{2\mu b_{(2\mu)} \sin 2\mu t}{\sin t} \quad (10)$$

where n_0 and n_e are the highest odd and even harmonics, respectively. Recalling the identities

$$\frac{\sin (2k+1)\theta}{\sin \theta} = 1 + 2 \sum_{j=1}^{k} \cos 2j\theta \quad (11)$$

and

$$\frac{\sin 2k\theta}{\sin \theta} = 2 \sum_{j=1}^{k} \cos (2j-1)\theta \quad (12)$$

and putting (11) and (12) into (10), there is obtained

$$g_m(t) = \frac{1}{E} \sum_{\mu=0}^{\mu=(n_0-1)/2} \left[(2\mu+1) b_{(2\mu+1)} \left(1 + 2 \sum_{j=1}^{j=\mu} \cos 2jt \right) \right]$$
$$+ \frac{2}{E} \sum_{\mu=1}^{\mu=n_e/2} \left[2\mu b_{(2\mu)} \sum_{j=1}^{j=\mu} \cos (2j-1)t \right]. \quad (13)$$

Eq. (13) corresponds to (7). Now, the coefficients of terms of like harmonic are chosen; these are called g_s.

For even values of $s > 0$ let $s = 2\sigma$

$$g_{(2\sigma)} = \frac{2}{E} \sum_{\mu=\sigma}^{\mu=(n_0-1)/2} (2\mu + 1) b_{(2\mu+1)} \tag{14}$$

and for odd values of s let $s = 2\sigma - 1$

$$g_{(2\sigma-1)} = \frac{2}{E} \sum_{\mu=\sigma}^{\mu=n_0/2} 2\mu b_{(2\mu)}. \tag{15}$$

Typical specific results would be as follows:

$$g_1 = \frac{2}{E}(2b_2 + 4b_4 + 6b_6 + \cdots)$$

$$g_2 = \frac{2}{E}(3b_3 + 5b_5 + 7b_7 + \cdots)$$

$$g_3 = \frac{2}{E}(4b_4 + 6b_6 + 8b_8 + \cdots)$$

etc.

Conclusion

Eq. (6) was developed to give the general relation for the coefficients of the power series expansion of a nonlinear device in terms of the Fourier series coefficients measured at the output. In (14) and (15) are relations giving harmonic conversion transconductance coefficients in terms of similarly measured Fourier series coefficients.

Leak Detection—Ultrasensitive Techniques Employing the Helium Leak Detector*

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Summary—**The development of closures for electron devices often requires that leaks far below the usual range of the helium mass spectrometer-type leak detector be localized and measured. It has been possible, by using new techniques, to extend the sensitivity of detection several orders of magnitude beyond the normal limit of the instrument. Two highly sensitive methods with similar techniques but separate sensitivity ranges are discussed.**

In seals similar to those of color television picture tube closures, it is noted that the character of small leaks is such that long times are required to reach a constant maximum rate of helium leakage. Methods for predicting this maximum rate from early observations are discussed. Methods for establishing procedures necessary for sensitivity requirements for any particular seal type are given. Time and sensitivity limitations are discussed.

Increases in the sensitivity of detection of the order of 40,000 are attained easily. Leaks as small as 1.5 × 10^{-10} μliters/second have been detected and measured in 22-inch rectangular color television picture tube closure seals. In one year, a leak of this size would only cause an increase in pressure in an ungettered tube of 10^{-7} mm Hg. This represents an increase in the sensitivity of detection of about one million.

Introduction

THE DESIGN of most color television picture tubes requires that the panel and funnel of the envelope be sealed together during the tube manufacturing process. Early in the development of the solder glass seal for glass envelopes (shown sandwiched between the panel and funnel in Fig. 1), it was realized that leaks large enough to cause operational failure of tubes were too small to be found with the helium leak detector when using normal techniques. The usual method of testing such seals involved measuring the pressure rise with time in the finished tube by using the electron gun as an ionization gauge. However, this method (known as gas ratio testing) has several shortcomings.

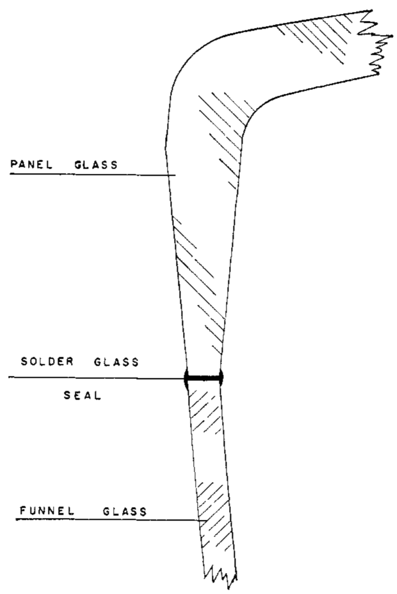

Fig. 1—Solder glass seal for glass color television picture tube envelopes.

1) It is difficult to determine if a pressure rise in a tube is the result of virtual leakage (due to outgassing) or the result of in-leakage.
2) It is difficult to localize in-leakage.

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