Composite Operational Amplifiers: Generation and Finite-Gain Applications

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Abstract —**A practical and effective general approach is presented for extending the useful operating frequencies and improving the performance of linear active networks realized using operational amplifiers (OA's). This is achieved by replacing each OA in the active network by a composite operational amplifier (CNOA) constructed using N OA's. The technique of generating the CNOA's for any given N is proposed. The realizations employing the CNOA are examined according to a stringent performance criterion satisfying such important properties as extended bandwidth, stability with one- and two-pole OA models, low sensitivity to the components and OA mismatch, and wide dynamic range. Several families of CNOA's, for $N = 2$, 3, and 4, are shown to satisfy the suggested performance criterion. In this contribution, the CNOA's applications in inverting, noninverting, and differential finite-gain amplifiers are given and shown theoretically and experimentally to compare favorably with the state-of-the-art realizations using the same number of OA's. Applications of the CNOA in inverting integrator and active filter realizations are presented in a companion contribution [32].**

I. Introduction

LINEAR ACTIVE circuits, namely, positive, negative, and differential finite-gain amplifiers, integrators, and active filters are usually realized with operational amplifiers (OA's) as the active elements. These active elements have frequency-dependent gains which restrict the operating frequencies of the linear active circuits. The operating frequencies are defined to be those frequencies for which the deviation of the actually obtained transfer function $T_a(s)$ of an active realization from its ideal value $T_i(s)$ (due to the OA's finite gain and frequency dependence) falls within a predetermined acceptable range. The frequency limitations due to the passive components are not addressed here. In practice, the passive components restrict operation in a higher frequency range relative to that of the OA.

For practical reasons, extending the useful bandwidth (BW) of the most commonly used linear active circuits has received the attention of many researchers in this field. This has resulted in many contributions, each dealing with the solution of this frequency limitation in specific applications [1]–[14]. Generally, three approaches were considered to minimize the dependence of the realization on the active

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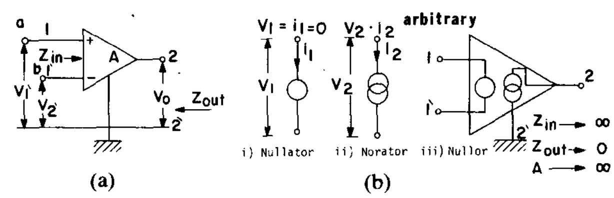


Fig. 1. (a) An operational amplifier (VCVS) and (b) its nullor representation.

device parameters and consequently its variations [15], [16]. In the first approach, for a given fixed number of OA's, the passive configuration in which the OA's are embedded (called the companion network) is carefully designed. In the second approach, an increased number of OA's are used to realize a given $T_i(s)$. This results in increased degrees of freedom in choosing the companion network. In the third approach, each OA in a given configuration is simply replaced by an OA that has improved characteristics, such as wider gain–bandwidth product (GBWP).

Recently, the authors suggested an approach using so-called composite operational amplifiers (CNOA's) that achieved a considerable performance improvement and bandwidth extension of almost all linear active networks used in signal processing and amplification for audio and video communications as well as instrumentation [17]–[20]. This has been verified by other researchers [21], [22]. In addition, the CNOA concept has proved to be useful in nonlinear and high-speed, high-accuracy applications, such as fast A/D and D/A conversion, digital communications, and switched capacitor filtering [23]–[26], [34]. The objective of this paper is to present a comprehensive treatment of the CNOA method and its application in linear active signal processing. In this technique, each OA is replaced by a composite operational amplifier (CNOA) [17]–[20] without modifying the topology of the companion network.

In Section II, the procedure for generating the CNOA's is presented. First, a general technique for the generation of a number of C2OA's ($N = 2$) using nullator–norator pairing [27]–[30] is described. Four of the C2OA's are found to meet useful performance criteria; these are retained for use in design. To further illustrate the generality of the proposed technique, the generation of the CNOA's

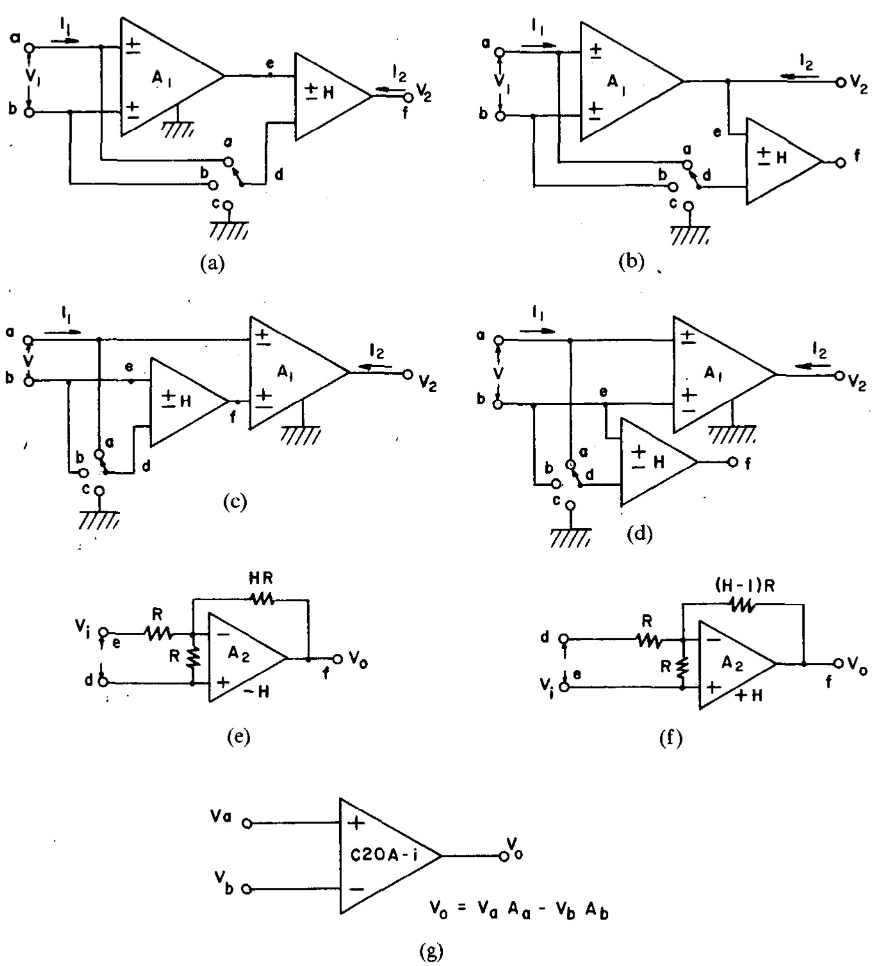


Fig. 2. (a)–(d) Four different networks for generating the composite operational amplifiers using two single OA's (C2OA's). (e) −H and (f) +H finite-gain amplifier realizations used in Fig. 4(a)–(d). (g) The composite operational amplifier (C2OA-*i*) symbol.

for $N>2$ is described. Sample results of C3OA's and C4OA's, which meet the above performance criteria, are presented.

Use of the proposed CNOA's in inverting and noninverting finite-gain applications is given in Section III. It is shown theoretically and experimentally that appreciable performance improvements are realized over the present state-of-the-art designs which utilize the same number of OA's.

II. Generation of Composite Operational Amplifiers (CNOA's) using *N* single OA's

A. Generation of the C2OA's ($N=2$)

An operational amplifier shown in Fig. 1(a) is a voltage-controlled voltage source (VCVS). In the ideal case, the input impedance $Z_{in} \to \infty$. This corresponds to the model shown in Fig. 1(b), which uses nullator and norator singular elements [27]–[30]. The ideal OA is replaced by a nullor which is described by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}. \quad (1)$$

The matrix in (1) is called the nullor chain transmission matrix of an ideal OA. In any physical circuit that contains N OA's, replacing each OA by a nullor results in a nullor equivalent network. The nullors can further be split into nullators and norators to yield a nullator–norator equivalent network.

Similarly, a nullator–norator equivalent network containing N nullators and N norators yields $N!$ nullor equivalent networks, since nullators and norators can be paired into nullors in an arbitrary manner.

Although the nullator (or norator) alone is not an admissible element for modeling a physical network, the nullor, like the infinite-gain controlled source, can be used for this purpose. The equivalence established is valid whether $A \to \infty$ or $A \to -\infty$, and so in practice a nullor can be replaced by a high-gain differential controlled source in two ways. In general, a nullor equivalent network containing N nullors corresponds to 2^n physical networks. Each of these $N!$ nullor networks yields a physical realization which has a different dependence on the nonideal active elements. This subject is well documented in the literature [27]–[30].

The procedure to generate the C2OA's is as follows. In the first step, a redundant amplifier of finite gain $\pm H$

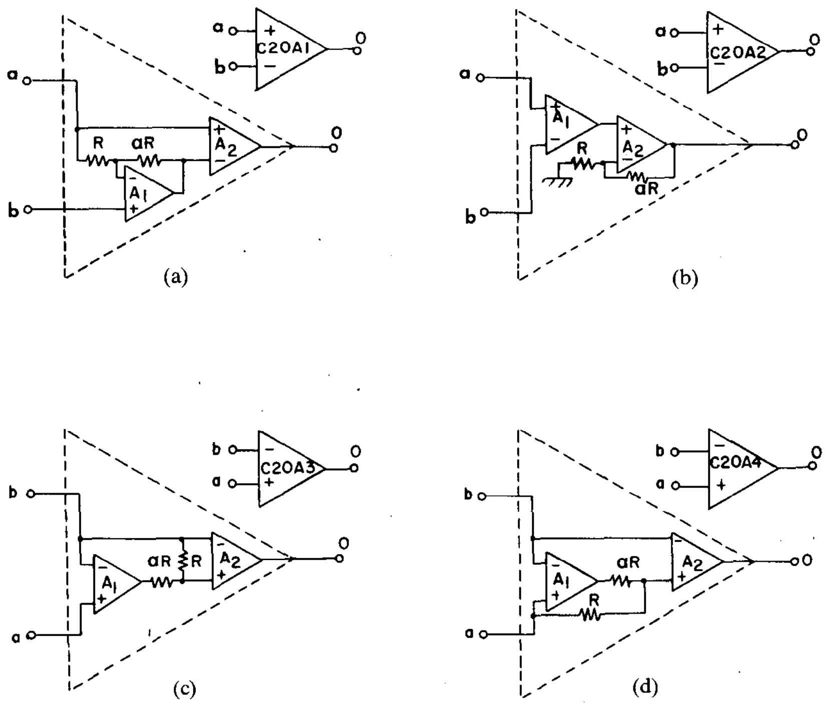


Fig. 3. The composite operational amplifiers (C2OA's). (a) C2OA-1. (b) C2OA-2. (c) C2OA-3. (d) C2OA-4.

(Fig. 2(e) and (f)) is combined with a single OA such that the chain matrix of the resulting two-amplifier network (assuming ideal amplifiers) corresponds to that of a nullor, as given in (1). That is, although each network contains two VCVS's, the overall two-port network realizes one VCVS. Six possible topologies can be obtained for each of the four networks shown in Fig. 2(a)–(d). Two topologies are obtained, one for $+H$ and the other for $-H$, for each position of the three-way switch, leading to six topologies per network. It is easy to show that 17 of the 24 topologies realize true nullors; i.e., no special stipulation on network elements or signals is required. Eight possible OA realizations can be obtained from each of these 17 topologies (nullor networks). This results in 136 composite operational amplifiers (C2OA's), each constructed using two single OA's. The resulting C2OA's, shown in Fig. 2(g), are examined according to the following performance criteria.

i) Let $A_a(s)$ and $A_b(s)$ denote the noninverting and inverting open-loop gains of each of the 136 C2OA's examined. The denominator polynomial coefficients of $A_a(s)$ and $A_b(s)$ should show no change in sign. This satisfies the necessary (but not sufficient) conditions for stability. Also, none of the numerator or denominator coefficients of $A_a(s)$ and $A_b(s)$ should be realized through differences. This eliminates the need for single OA's with matched GBWP's and results in low sensitivity of the C2OA with respect to its components.

ii) The external three-terminal performance of the C2OA should resemble as closely as possible that of the single OA.

iii) No right-half s-plane (RHS) zeros due to the single OA pole should be allowed in the closed-loop gains of the C2OA's (for minimum phase shifts).

iv) The resulting input–output relationship $T_a(s)$ in the applications considered should have extended frequency operation with minimum gain and phase deviation from the ideal $T_i(s)$. The improvement should be sufficient to justify the increased number of OA's.

Four C2OA's referred to as C2OA-1, C2OA-2, C2OA-3 and C2OA-4, of the 136 examined are found to meet these performance criteria, and are shown in Fig. 3.

It is interesting to note that a special case of C2OA-3 can be derived from the transistor Darlington pair [31], where the norators are both at ac ground in the Darlington pair enabling an OA realization.

The open-loop gain of the single OA's used in the modeling of the C2OA's (assuming a single-pole model) is

$$A_i = \frac{A_{oi}\omega_{Li}}{\omega_{Li}+s} = \frac{\omega_i}{s+\omega_{Li}}, \qquad i=1 \text{ or } 2 \tag{2}$$

where A_{oi}, ω_{Li} and ω_i are the dc open-loop gain, the 3-dB bandwidth, and the GBWP of the ith single OA, respectively.

It can be easily shown that the open-loop input–output relationships for the C2OA-1 to C2OA-4 are given by

$$V_{oi} = V_a A_{ai}(s) - V_b A_{bi}(s) \qquad (i=1\cdots 4)$$

where for C2OA-1

$$V_{o1} = V_a \frac{A_2(1+A_1)(1+\alpha)}{A_1+(1+\alpha)} - V_b \frac{A_1 A_2(1+\alpha)}{A_1+(1+\alpha)} \tag{3}$$

for C2OA-2

$$V_{o2} = V_a \frac{A_1 A_2(1+\alpha)}{A_2+(1+\alpha)} - V_b \frac{A_1 A_2(1+\alpha)}{A_2+(1+\alpha)} \tag{4}$$

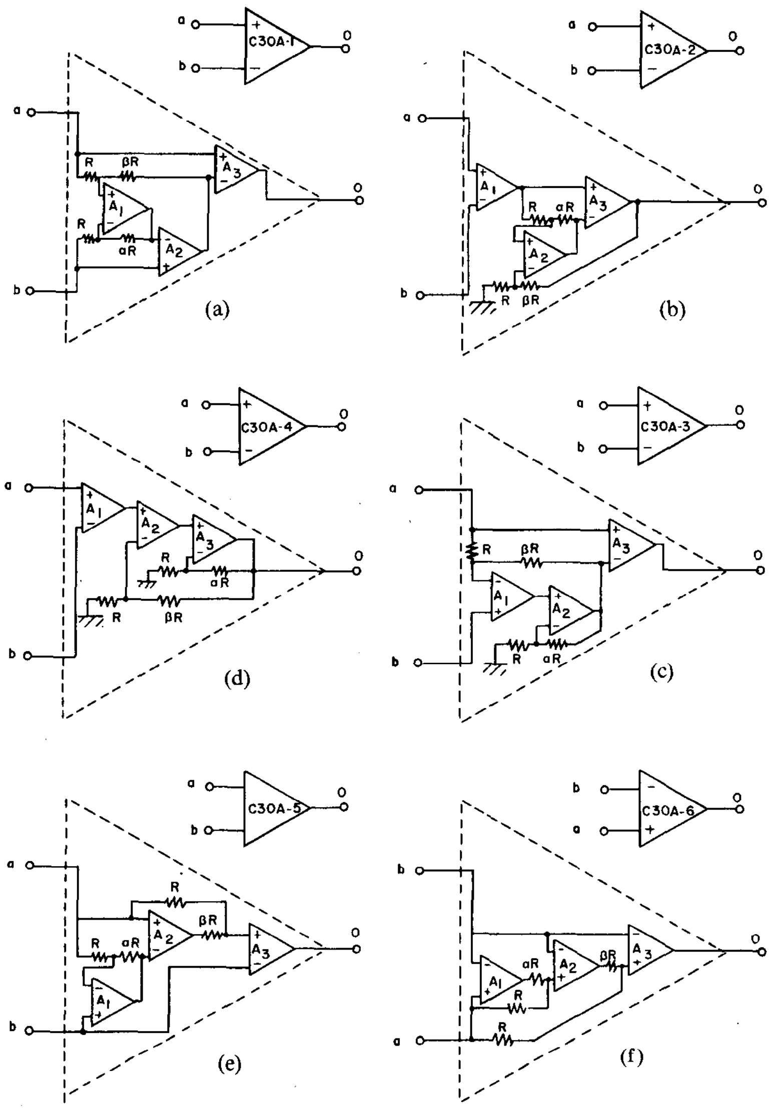


Fig. 4. The composite operational amplifiers (C3OA's). (a) C3OA-1. (b) C3OA-2. (c) C3OA-3. (d) C3OA-4. (e) C3OA-5. (f) C3OA-6.

for C2OA-3

$$V_{o3}=V_a\frac{A_1A_2}{(1+\alpha)}-V_b\frac{A_2(1+A_1)}{(1+\alpha)} \tag{5}$$

and for C2OA-4

$$V_{o4}=V_a\frac{A_2(A_1+\alpha)}{(1+\alpha)}-V_b\frac{A_2[A_1+(1+\alpha)]}{(1+\alpha)} \tag{6}$$

where α is a resistor ratio, as illustrated in Fig. 3.

Assuming identical OA's, i.e.,

$$A_{o1}=A_{o2}=A_o \text{ and } \omega_1=\omega_2=\omega_i$$

it is interesting to examine the open-loop gains given by (3)–(6) in the single-ended inverting application, i.e., when $V_a=0$.

For C2OA-1 and C2OA-2, the open-loop dc gain A_{oC1} is given by

$$A_{oC1}=\frac{A_o(1+\alpha)}{1+(1+\alpha)/A_o}\simeq A_o(1+\alpha) \qquad \text{for } (1+\alpha)\ll A_o. \tag{7a}$$

From (2) and (7a), the composite amplifier has a single-pole rolloff from ω_i/A_o to $\omega_i/(1+\alpha)$, where the second pole occurs. As α increases, the dc gain increases while the frequency of the second pole decreases.

Also, from (5) and (6), both the C2OA-3 and the C2OA-4 have an open-loop dc gain given by

$$A_{oC2}=\frac{A_o^2}{(1+\alpha)}. \tag{7b}$$

From (2) and (7b), A_{oC2} has double poles (12-dB/octave) at ω_i/A_o, and as α increases the dc gain decreases without affecting the location of the second pole.

Only the C2OA-2 has identical expressions for the positive and negative open-loop gains A_a and A_b. Thus, common-mode rejection ratio (CMRR) problems should not be encountered using C2OA-2, even for relatively large common-mode signal applications. From (3), (5), and (6), the CMRR of the C2OA-1 and C2OA-3 is $(A_{o1}+1/2)$, while that of the C2OA-4 is $(A_{o1}+\alpha+1/2)$. For single-ended applications (small common-mode signal), no prob-

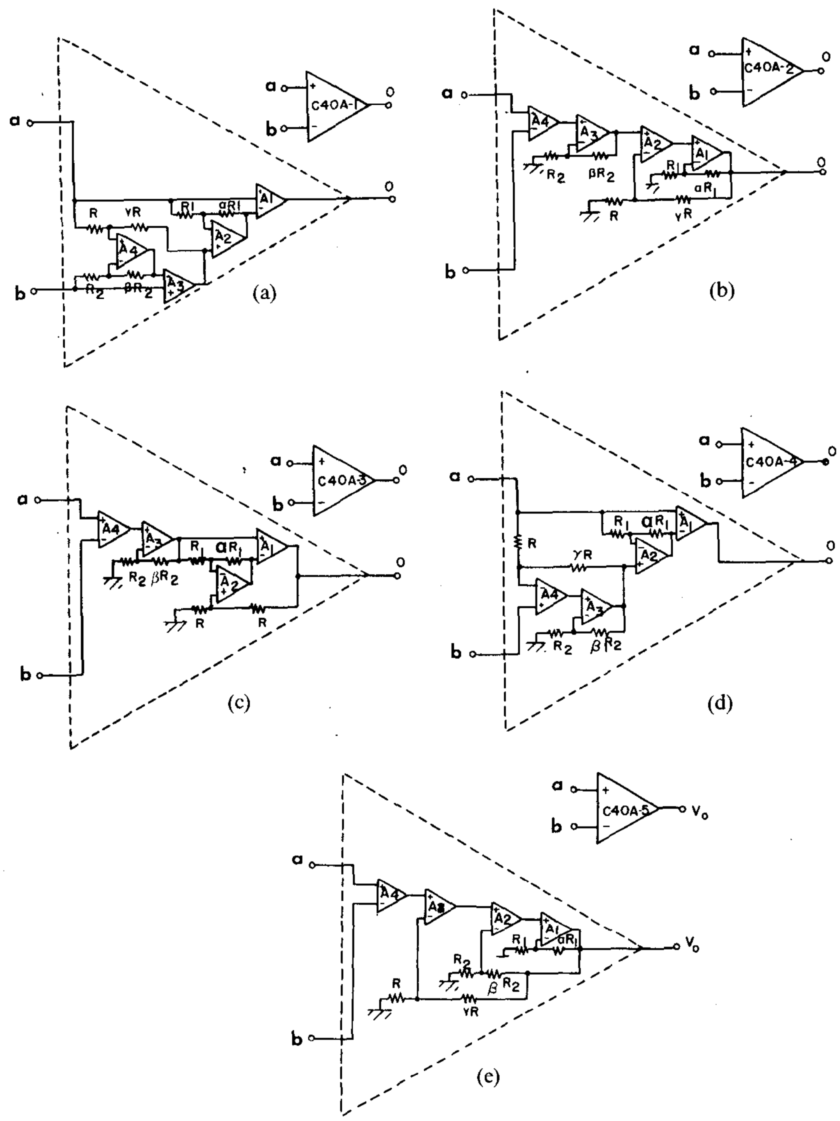


Fig. 5. The composite operational amplifiers (C4OA's). (a) C4OA-1. (b) C4OA-2. (c) C4OA-3. (d) C4OA-4. (e) C4OA-5.

lems are anticipated in using the C2OA-1, C2OA-3, or C2OA-4, as verified experimentally later.

It is easy to show that the voltage swing at the first OA(A_1) output, which is an internal node in each of C2OA-1 to C2OA-4, is always less than the output voltage V_o. Hence, the dynamic range is determined by the voltage swing of the output voltage V_o. Consequently, no dynamic-range reduction of V_o or harmonic distortion problems should arise.

B. Generation of CNOA's ($N > 2$)

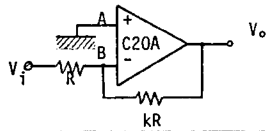
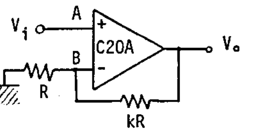
Following an analogous approach, CNOA's for $N > 2$ can be generated for extending the operating frequencies at the expense of additional amplifiers. The CNOA's can be obtained in two different ways. The first approach starts from the basic single OA with additional redundant amplifiers. Then, nullator–norator pairing is used as described in Section II-A.

In the second approach, which is used here, the C2OA's are used as single OA replacements in the C2OA structure. Although this second approach is not exhaustive, it will be shown to yield excellent results. Hence, C3OA's are obtained by starting with one of the proposed C2OA's and replacing one of its single OA's by any of the C2OA's in Fig. 3. Thirty-two possible combinations of C3OA's can be obtained using the four proposed C2OA's.

Similarly, C4OA's are generated here by replacing each of the single OA's in a C2OA with any of the C2OA's or by replacing one of the single OA's in C3OA's with a C2OA. This results in many possible combinations of C4OA's. The process can be continued (by using C2OA's, C3OA's, and C4OA's) to obtain CNOA's for any number N. N should be limited in practice; the increased complexity is expected to give rise to practical problems in spite of the advantages of an extended operating range.

Samples of C3OA and C4OA novel designs, which meet the performance criteria described in Section II-A, are shown in Figs. 4 and 5. The open-loop expressions of these C3OA's and C4OA's, as well as others, can be found elsewhere [33].

TABLE I
NEGATIVE AND POSITIVE FINITE GAINS V_o/V_i USING THE C2OA'S

C2OA-i	Negative Finite Gain Trans. Function (Ta)	Positive Finite Gain Trans. Function (Ta)	ω_p	Q_p
C2OA-1	$T_i \dfrac{1}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$T_i \dfrac{(1+S/\omega_1)}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$\sqrt{\dfrac{\omega_1\omega_2}{1+k}}$	$\dfrac{(1+\alpha)}{\sqrt{1+k}}\sqrt{\dfrac{\omega_2}{\omega_1}}$
C2OA-2	$T_i \dfrac{1}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$T_i \dfrac{1}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$\sqrt{\dfrac{\omega_1\omega_2}{1+k}}$	$\dfrac{(1+\alpha)}{\sqrt{1+k}}\sqrt{\dfrac{\omega_1}{\omega_2}}$
* C2OA-3	$T_i \dfrac{(1+S/\omega_1)}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$T_i \dfrac{1}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$\sqrt{\dfrac{\omega_1\omega_2}{(1+k)(1+\alpha)}}$	$\sqrt{\dfrac{(1+k)(1+\alpha).\omega_1}{\omega_2}}$
C2OA-4	$T_i \dfrac{[1+(1+\alpha)S/\omega_1]}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$T_i \dfrac{(1+\alpha S/\omega_1)}{1+(S/\omega_pQ_p)+(S^2/\omega_p^2)}$	$\sqrt{\dfrac{\omega_1\omega_2}{(1+k)(1+\alpha)}}$	$\sqrt{\dfrac{(1+k)}{(1+\alpha)}\cdot\dfrac{\omega_1}{\omega_2}}$
				
	$\frac{V_o}{V_i} = -k = T_i$	$\frac{V_o}{V_i} = (1+k) = T_i$	T_i (ideal Transfer Function)	

$^*\alpha R_1 \ll kR$ (for maximum ω_p).

III. REALIZATION OF POSITIVE, NEGATIVE, AND DIFFERENTIAL FINITE-GAIN AMPLIFIERS

A. Finite Gain Amplifiers Using the Proposed C2OA's

The application of the four proposed C2OA's in positive and negative finite-gain amplification is given in Table I. For the differential finite-gain realization, often referred to as the instrumentation amplifier shown in Fig 6, C2OA-2 is used. The network in Fig. 6, when each single OA is modeled by (2), can be shown to have the input–output relationship given by

$$V_o = T_{i1}\left[\frac{1}{1+s/\omega_pQ_p+s^2/\omega_p^2}\right]V_1 + T_{i2}\left[\frac{1}{1+s/\omega_pQ_p+s^2/\omega_p^2}\right]V_2 \quad (8)$$

where

$$T_{i1} = X(1+k)/(1+X)$$
$$T_{i2} = -k$$
$$\omega_p = \sqrt{\frac{\omega_1\omega_2}{1+k}}$$
$$Q_p = \sqrt{\frac{\omega_1}{\omega_2(1+k)}}\cdot(1+\alpha).$$

For the differential gain application given above and the finite-gain applications in Table III, the actual input–output relationship T_a has the form

$$T_a = T_i\cdot\frac{N}{D} \quad (9)$$

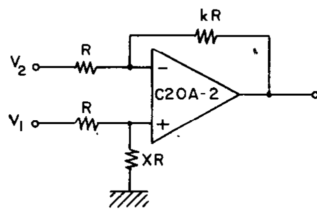


Fig. 6. Application of the C2OA-2 as a differential finite-gain amplifier.

where T_i = the transfer function realized assuming ideal OA's and

$$N = 1+as = 1+\frac{s}{\omega_z} \qquad (a \text{ is zero } (\omega_z\to\infty) \text{ in some cases})$$

$$D = 1+b_1s+b_2s^2 = 1+(s/\omega_pQ_p)+(s^2/\omega_p^2).$$

Thus, N/D indicates the amplitude and phase deviation of T_a from T_i. Also, b_1 and b_2 determine the stability of T_a, while a, b_1, and b_2, and consequently ω_z, ω_p, and Q_p, are functions of the circuit parameters ω_1, ω_2, and α. None of the a and b coefficients is realized through differences, which guarantees the low sensitivity of T_a, ω_z, ω_p, and Q_p to the circuit parameters. On the other hand, the b coefficients are always positive (assuming a single-pole OA model), which is necessary for the stability of the transfer function. From Table I, a mismatch of ±5 percent in ω_1 and ω_2 results in a ±5-percent change in ω_p and a ±2.5-percent change in Q_p. Hence, single OA's with mismatched gain–bandwidth products within practical ranges can be used without appreciably affecting the stability or the sensitivity of the finite-gain realizations.

TABLE II
VALUES OF α FOR MAXIMALLY FLAT AND FOR $Q_p = 1$ FINITE-GAIN REALIZATIONS USING C2OA'S AND THE CORRESPONDING BANDWIDTH AND STABILITY CONDITIONS

C2OA-i	$1+\alpha$	Q_p	ω_p	Stability Condition for α used
C2OA-1 &	$\sqrt{1+k}$	1	$\frac{\omega_i}{\sqrt{1+k}}$	Satisfied
C2OA-2	$\sqrt{\frac{1+k}{2}}$	$\frac{1}{\sqrt{2}}$	(independent of α)	Satisfied
C2OA-3	0	$Q_{p_{min}} = \sqrt{1+k}$	$\frac{\omega_i}{\sqrt{1+k}}$	Unsatisfied
C2OA-4	(1+k)	1	$\frac{\omega_i}{1+k}$	Unsatisfied
	2(1+k)	$\frac{1}{\sqrt{2}}$	$\frac{\omega_i}{\sqrt{2}(1+k)}$	Unsatisfied

1) Effect of the Single OA's Second Pole on the Stability of the C2OA's Finite-Gain Realizations: In the following, the stability properties of the positive and negative finite-gain amplifier realizations using a two-pole, open-loop model of the single OA's is studied. If the dc gains of the first and second OA's (A_1 and A_2, respectively) are assumed to be equal, this greatly simplifies the analysis without affecting the reliability of the conclusions. This is due to the absence of gain difference terms in all the gain expressions obtained, as seen from (3)–(6), (8), and Table I. Let

$$A = A_1 = A_2$$

where $1/A$ is given by

$$\frac{1}{A} = \left(1 + \frac{s}{\omega_h}\right)\left(\frac{s}{A_o\omega_L} + \frac{1}{A_o}\right) \tag{10}$$

and $\omega_h \gg \omega_L$.

By invoking the Routh–Hurwitz stability criterion [16], the necessary and sufficient condition for stability using the C2OA-1 or C2OA-2 [33] is found to be

$$(1+\alpha) < (1+k)/2 \tag{11}$$

while for the C2OA-3 the condition [33] is found to be

$$(1+\alpha) > \sqrt{(1+k)}\,. \tag{12}$$

Finally, for the C2OA-4 the condition [33] is given by

$$(1+\alpha) > 4(1+k). \tag{13}$$

Upon examining (11), (12), and (13), one finds that α imposed by the stability conditions (not necessarily BW conditions) is physically realizable for all k. In practice, α should be chosen in the stable range for a given k that results in the best realizable value of Q_p and ω_p. Table II gives the values of α required to yield $Q_p = 1/\sqrt{2}$ and $Q_p = 1$ for the realizations in Table I. The useful BW's of the different finite-gain amplifiers can be obtained by comparing the ω_p's in Table II. As ω_p increases for a fixed Q_p, both amplitude and phase deviations of T_a from T_i at a given frequency ω ($\omega < \omega_p$) decrease. It can be easily shown that the differential finite-gain amplifier in Fig. 6 has similar excellent bandwith and stability properties as those obtained for C2OA-2 in positive and negative finite-gain applications. *In conclusion, it is clear that the C2OA-1 and C2OA-2 are the most attractive configurations in finite-gain applications from BW and stability considerations.* It should be noted that some special cases of finite-gain amplifiers using C2OA's have been reported in the literature [1]–[3], [5], [6], [8] and cited for their improved performance.

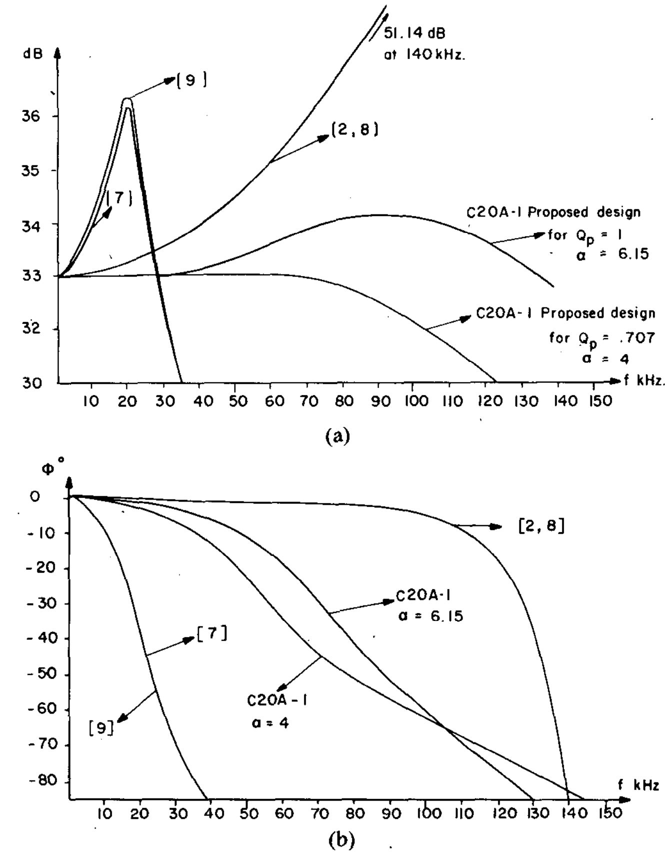


Fig. 7. Theoretical responses of the negative finite-gain amplifiers using C2OA-1 and the existing two-OA realizations (assuming OA GBWP = 1 MHz). (a) Frequency response of negative finite-gain amplifiers. (b) Phase response of negative finite-gain amplifiers.

2) Comparisons of the Proposed C2OA's Finite-Gain Realizations with Others: The BW of a finite-gain amplifier realized using a single OA shrinks approximately by a multiplying factor $1/k$ relative to its unity gain 3-dB BW (ω_i). Also, the optimum maximally flat 3-dB BW using a cascade of two (single-OA realization) finite-gain amplifiers is obtained when each amplifier has a gain $\sqrt{k}$ to realize an overall gain k. The resulting BW shrinks by $\sqrt{0.44}/\sqrt{k} = 0.66/\sqrt{k}$ relative to ω_i [31]. The C2OA-1 and C2OA-2 circuit BW's can be designed to shrink by only a factor of $\simeq 1/\sqrt{k}$ for $Q_p = 0.707$ (maximally flat) and greater than $1/\sqrt{k}$ for $Q_p = 1$ ($k \gg 1$); see Table II. In addition, the C2OA's require only two accurate gain-

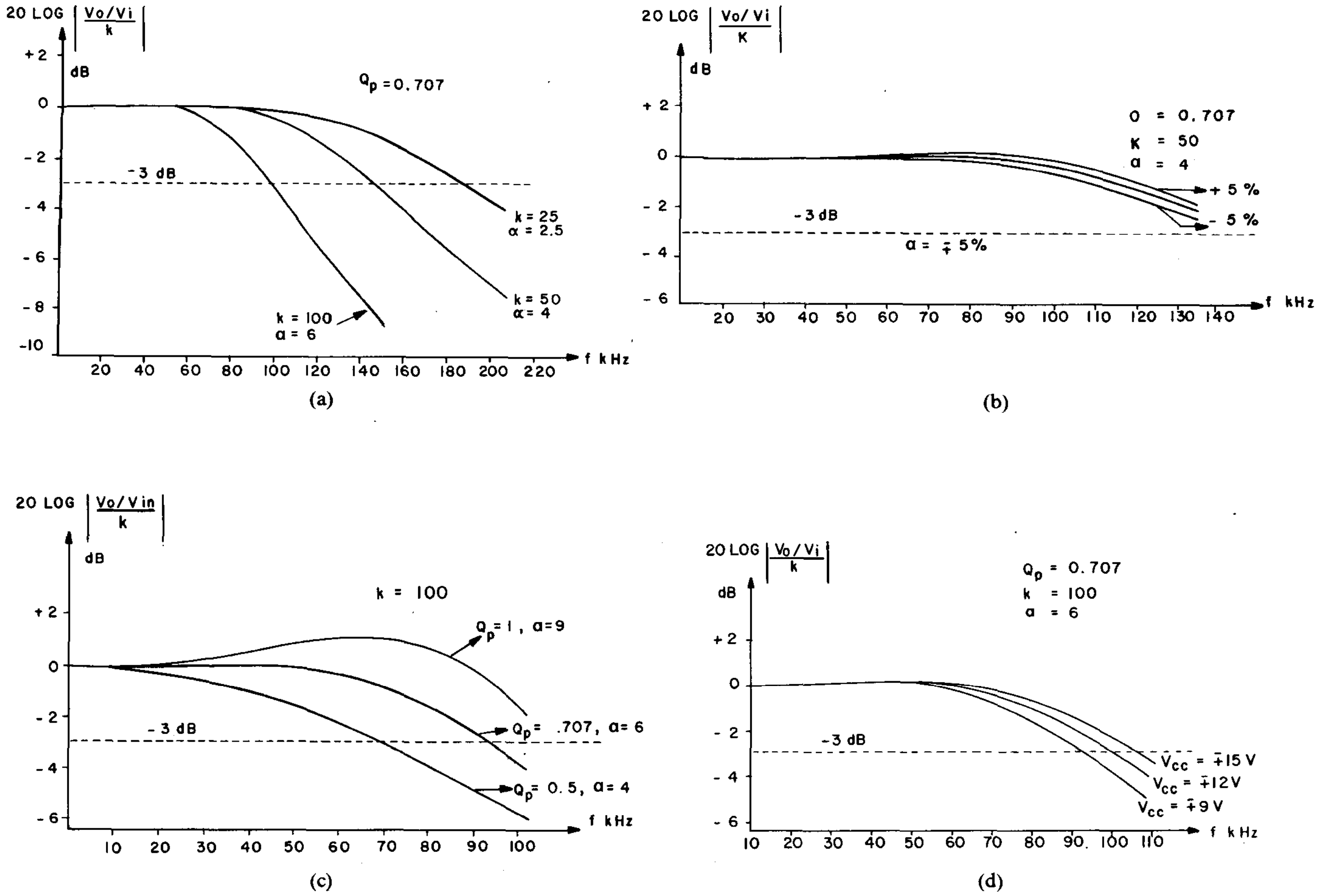


Fig. 8. Experimental results using C2OA-1 in negative gain applications. (a) ($Q_P = 0.707$) Maximally flat closed-loop gain $= -25, -50, -100$ (LM747 OP AMPS). (b) Effect of compensation resistor-ratio variation by ∓ 5 percent (LM747 OP AMPS). (c) Effect of active compensation on extending the bandwidth (LM747 OP AMPS). (d) Effect of power supply variation from ∓ 9 V to $\mp$ 15 V on the closed-loop gain for $k = 100$ (LM 747 OP AMPS).

determining components, compared with four in the cascade realization.

To further illustrate the usefulness of the C2OA's, one of the common applications considered in this paper is chosen, namely, negative finite-gain amplification. The performance of the C2OA-1 in this application is illustrated and compared in Fig. 7 with some of the most recently published negative finite-gain realizations which utilize a similar number of OA's. The results shown in Fig. 7 are for nominal gains $\gg 1$ for practical reasons since an increase in k decreases the useful bandwidth, so that extending the range of operating frequencies becomes more important. The proposed realizations are seen to be far superior in both amplitude and phase responses relative to those reported in [7] and [9]. Upon examining those of [2] and [8], one may erroneously conclude that, in spite of their inferior amplitude characteristics, they have better phase response. In fact, the realizations of [2] and [8], in contrast with the proposed ones, can be easily shown in theory to be unstable for all useful values of closed-loop gains, due to the second OA pole. This has been verified experimentally as well. Indeed, the results in Fig. 7 show clearly the excellent gain and phase performance of the proposed realizations.

3) Experimental Results Using C2OA's in Finite-Gain Applications: Experimental results of negative finite-gain amplifier realizations are given in Fig. 8(a) and (b) using the C2OA-1 of Fig. 3(a). LM747's with a GBWP ranging from 1 to 1.5 MHz were used to implement the C2OA's in this section as well as in the experiments throughout this work. The stability and low sensitivity to the power supply and to the active compensation resistor variations are examined as shown in Fig. 8(c) and (d). Comparing the results in Fig. 8 with those obtainable using single-amplifier realizations illustrates the considerable improvement in the useful BW without sacrificing any of the single-OA desirable features, namely, its low sensitivity to circuit elements and power supply variations, stability, and versatility.

B. Finite-Gain Applications Using C3OA's and C4OA's

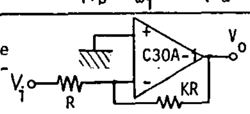
Wide-band positive, negative, and differential composite amplifiers can be designed using the CNOA structures proposed in Section II-B and shown in Figs. 4 and 5.

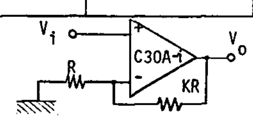
Positive and negative finite-gain expressions for C3OA-1 through C3OA-6 are given in Table III, while those for C4OA-1 through C4OA-4 can be found elsewhere [33]. In the finite-gain expressions of these C3OA's and C4OA's,

TABLE III
NEGATIVE AND POSITIVE FINITE GAINS V_o/V_i USING THE C3OA's

C3OA-i	Finite Gain Transfer Function		Stability Condition
C3OA-1	$\frac{V_o}{V_i} = \frac{-k(1+\frac{S}{\omega_1})}{1+(1+\frac{1+k}{1+\beta})\frac{S}{\omega_1}+(\frac{1+k}{1+\alpha})\frac{S^2}{\omega_2\omega_3}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Negative Finite Gain Trans. Func.	$(1+k)>\alpha(1+\beta)$
	$\frac{V_o}{V_i} = \frac{(1+k)(1+\frac{1}{1+\alpha}\frac{S}{\omega_3}+\frac{S^2}{\omega_1\omega_2})}{1+(1+\frac{1+k}{1+\beta})\frac{S}{\omega_1}+(\frac{1+k}{1+\alpha})\frac{S^2}{\omega_2\omega_3}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Positive Finite Gain Trans. Func.	
C3OA-2	$\frac{V_o}{V_i} = \frac{-K(1+S/\omega_2)}{1+[\frac{1}{\omega_2}+\frac{1+k}{\omega_1(1+\beta)}]S+\frac{(1+k)}{(1+\alpha)}\frac{S^2}{\omega_1\omega_2}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Negative Finite Gain Trans. Func.	$(1+k)>\alpha(1+\beta)$
	$\frac{V_o}{V_i} = \frac{(1+K)(1+S/\omega_2)}{1+[\frac{1}{\omega_2}+\frac{1+k}{\omega_1(1+\beta)}]S+\frac{(1+k)}{(1+\alpha)}\frac{S^2}{\omega_1\omega_2}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Positive Finite Gain Trans. Func.	
C3OA-3	$\frac{V_o}{V_i} = \frac{-K}{1+(\frac{1+k}{1+\beta})\frac{S}{\omega_3}+(\frac{1+k}{1+\alpha})\frac{S^2}{\omega_1\omega_3}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Negative Finite Gain Trans. Func.	$(1+k)>(1+\alpha)(1+\beta)$
	$\frac{V_o}{V_i} = \frac{(1+K)(1+\frac{1}{1+\alpha}\frac{S}{\omega_1}+\frac{S^2}{\omega_1\omega_2})}{1+(\frac{1+k}{1+\beta})\frac{S}{\omega_3}+(\frac{1+k}{1+\alpha})\frac{S^2}{\omega_1\omega_2}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Positive Finite Gain Trans. Func.	
C3OA-4	$\frac{V_o}{V_i} = \frac{-K}{1+(\frac{1+k}{1+\beta})\frac{S}{\omega_1}+(\frac{1+k}{1+\alpha})\frac{S^2}{\omega_1\omega_2}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Negative Finite Gain Trans. Func.	$(1+k)>(1+\alpha)(1+\beta)$
	$\frac{V_o}{V_i} = \frac{(1+K)}{1+(\frac{1+k}{1+\beta})\frac{S}{\omega_1}+(\frac{1+k}{1+\alpha})\frac{S^2}{\omega_1\omega_2}+(1+k)\frac{S^3}{\omega_1\omega_2\omega_3}}$	Positive Finite Gain Trans. Func.	

Negative Finite Gain Configuration





Positive Finite Gain Configuration

no terms containing differences are encountered; thus, low coefficient sensitivities are obtained and reasonable OA mismatch is tolerated. Also, all the denominator coefficients are positive, which is necessary for stability. Applying the same technique used in Sections II and III-A, it can be shown that the resistor ratios α, β, and γ can be chosen to extend the BW and to satisfy the necessary stability conditions assuming single-pole OA models.

1) Comparisons of the Proposed C3OA's and C4OA's Finite-Gain Realizations with Others: The optimum maximally flat 3-dB BW using three (four) single-OA finite-gain building blocks is obtained by cascading three (four) identical blocks, each with gain $k^{1/3}(k^{1/4})$ to realize an overall gain k. The overall BW shrinks by a multiplying factor $0.51/k^{1/3}(0.435/k^{1/4})$ relative to ω_i [31]. The BW of the new proposed C3OA (C4OA) circuits are found to shrink by only a factor $1/k^{1/3}(1/k^{1/4})$ $(k \gg 1)$.

Maximally flat response (Butterworth) as well as Chebyshev characteristics, using CNOA's, can be achieved by controlling the resistor ratios α, β, and γ while still satisfying the stability conditions. Computer plots of the C3OA-1 and C4OA-1 transfer functions in the positive and negative finite-gain configurations are given in Figs. 9 and 10 for gains of 100. From Figs. 9 and 10, it is seen that the 3-dB BW available from these C3OA (C4OA) finite-gain amplifiers implemented using 1-MHz single OA's corresponds to the BW attainable from a single OA with zero-dB BW in excess of 25 (35) MHz! The performance of the C3OA-1 is compared with the performance of recently published three-OA realizations, called the zero second derivative (ZSD) amplifiers, proposed in [10]. A positive finite gain of 38.7 is chosen to permit direct comparison with the theoretical results previously published in [10]. For practical reasons, both ZSD amplifiers are designed such that the stability condition, using Routh's test on the third-order denominator coefficients, is exceeded by a margin of 10 percent. The best theoretical results using the ZSD are obtainable with the minimum stability margin to allow for maximum bandwidth, i.e., $(r_1-1)=1.1\ \beta k$. Fig. 11(a) and (b) shows the theoretical magnitude and phase characteristics of the ZSD amplifiers, as well as the C3OA-1 and C4OA-1 amplifiers (which satisfy the stability constraints), with positive finite gain of 38.7, for different compensation values. The figure depicts the extended frequency range of operation attainable with these C3OA and C4OA designs over the ZSD realization [10], or a realization which uses three (four) cascaded single-OA finite-gain stages. It is interesting to note that the condition for stability of the amplifier shown in Table III using the C3OA-1 is

$$\alpha < \frac{1+k}{1+\beta}$$

where $k = 38.7$.

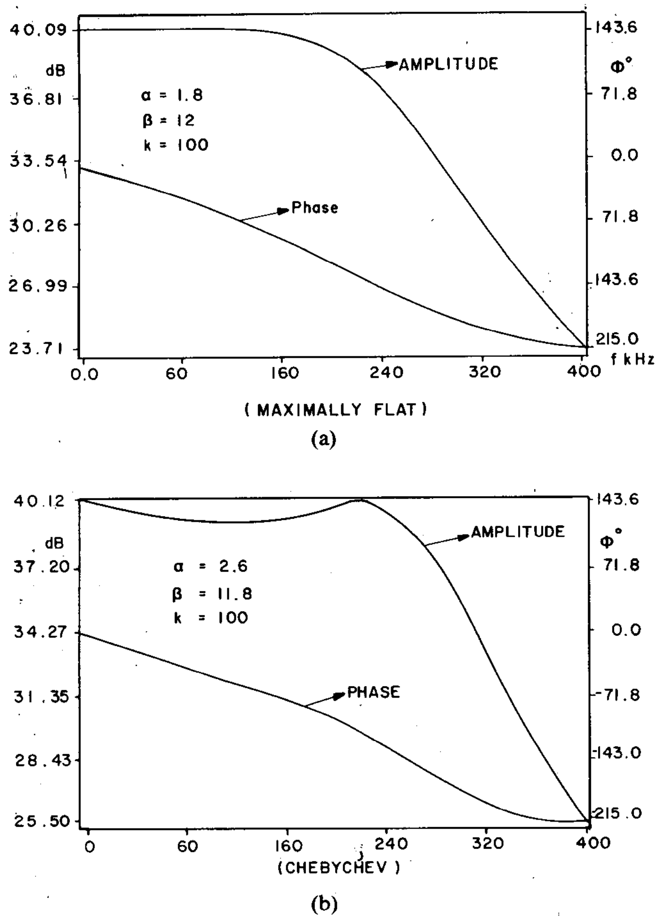


Fig. 9. Computer plots (magnitude and phase) of the C3OA-1 transfer function for gain 100. (a) Computed frequency response (amplitude and phase) of the positive finite-gain amplifier ($k=100$) using C3OA-1 (single OA GBWP = 1 MHz). (b) Computed frequency response (amplitude and phase) of the positive finite-gain amplifier ($k=100$) using C3OA-1 (single OA GBWP = 1 MHz).

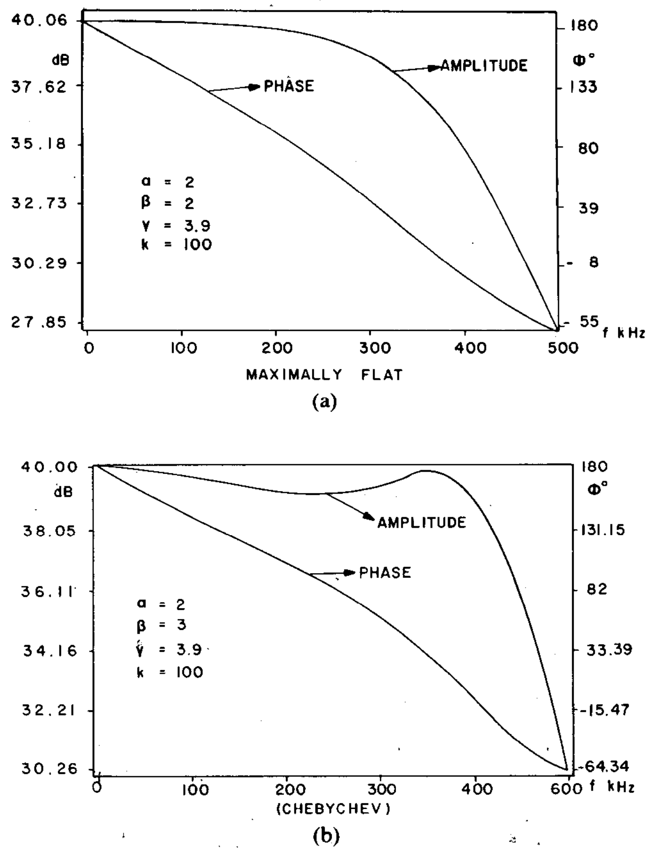


Fig. 10. Computer plots (magnitude and phase) of the C4OA-1 transfer function for gain 100. (a) Maximally flat computed frequency response (amplitude and phase) of the negative finite-gain amplifier ($k=100$) using C4OA-1 (single OA GBWP = 1 MHz). (b) Chebyshev computed frequency response (amplitude and phase) of the negative finite-gain amplifier ($k=100$) using C4OA-1 (single OA GBWP = 1 MHz).

This is satisfied by a wide margin in the C3OA-1 responses in Fig. 11(a) and (b) for both the maximally flat response and the Chebyshev response. All of these finite-gain designs have the same attractive dynamic-range, stability, and low-sensitivity properties as the C2OA designs in Section III-A. Also, it is interesting to note that some of the finite-gain designs presented here (C3OA-2, C3OA-4, C4OA-2, C4OA-3, and C4OA-5) have identical N/D multiplying factors in the positive and negative gain applications, which makes them suitable in differential gain applications.

2) Experimental Results Using C3OA's and C4OA's in Finite-Gain Applications: Only sample experimental results using the C3OA-1 and C4OA-1 are given to illustrate the performance. Exhaustive test results are documented in [33]. Fig. 12 gives the experimental results using the C3OA-1 in positive finite-gain applications of 38.7. The computer frequency response plots of the C4OA-1 negative finite-gain realization in Fig. 10 closely agree with the experimental results of Fig. 13.

The stability and low sensitivity to power supply as well as to the active compensation resistor variations were verified [33].

IV. Conclusions

A new approach is presented for extending the useful operating frequency range in a wide variety of linear active networks which utilize OA's. The extended BW is achieved by replacing each of the single OA's in the active realization by a composite OA (CNOA). The application of the CNOA's in finite-gain amplifiers is also given.

A systematic procedure is given for the generation of the CNOA's. Each CNOA is constructed using N single OA's and $2(N-1)$ active compensating low-spread and low-accuracy resistors, resulting in $(N-1)$ resistor ratios. The CNOA is versatile since it has three external terminals that correspond to those of a single OA. The suggested generation method gives rise to a large number of CNOA's for a given N. For $N=2$, 3, and 4, CNOA's are generated and examined according to a stringent performance criterion that considers stability, sensitivity, dynamic range, CMRR, BW, and the GBWP mismatch effect of single OA's. Several of the CNOA's, namely the C2OA-1 to C2OA-4, C3OA-1 to C3OA-6, and C4OA-1 to C4OA-5, meet the performance criterion and have been found to be very useful in practice. In these CNOA's, simple resistor ratios can be used advantageously to reduce the deviation of the

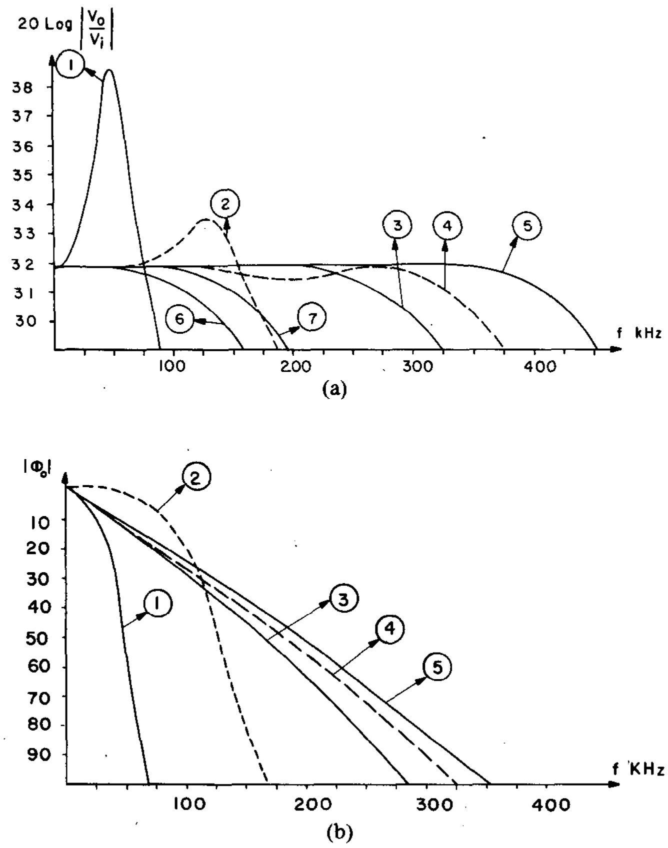


Fig. 11. Theoretical results for C3OA-5, C4OA-5, and [10] for positive finite-gain applications (gain $k = 38.7$). ①: [10] $r = 1.1$, $\beta = 10^{-8}$ (stable with min. margin); ②: [10] $r = 2$, $\beta = 0.2$ (unstable); ③: C3OA-5 $\alpha = 1.9$, $\beta = 5.4$; ④: C3OA-5 $\alpha = 1$, $\beta = 6.4$; ⑤: C4OA-5 $\alpha = 0.6$, $\beta = 2.4$, $\gamma = 12.5$; ⑥: cascade of three single-OA finite-gain stages, each of gain $(38.7)^{1/3}$; ⑦: cascade of four single-OA finite-gain stages, each of gain $(38.7)^{1/4}$. (a) Theoretical amplitude responses of C3OA-5, C4OA-5, and [10] for positive finite-gain applications. (b) Theoretical phase responses of C3OA-5, C4OA-5, and [10] in positive finite-gain applications.

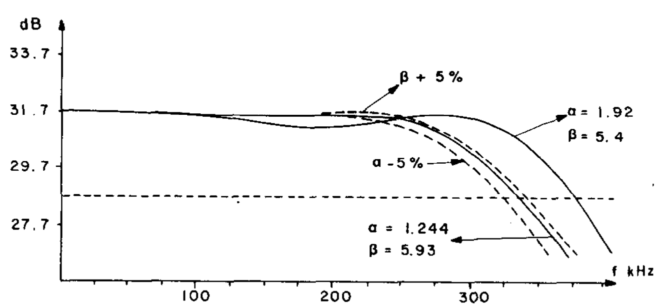


Fig. 12. Experimental results of C3OA-1 in positive finite-gain application, for a gain of 38.7, and the effect of variation of the compensating resistor ratios α and β (LM747 OP AMPS).

overall active realization's response from the ideal while guaranteeing stability.

Finite-gain applications utilizing the proposed CNOA's are shown both theoretically and experimentally to be stable and to exhibit wide dynamic range and low sensitivity. Comparisons with the the state-of-the-art realizations using similar numbers of OA's in these applications show the appreciable improvement of the realizations obtained

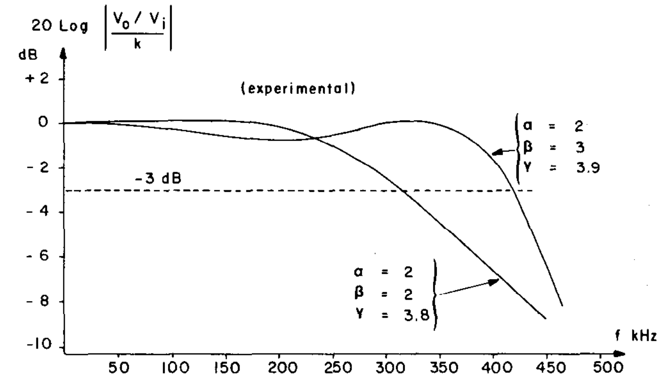


Fig. 13. The effect of active compensation on extending the bandwidth of the C4OA-1 (using LM747 OP AMPS).

using the proposed CNOA's with respect to stability and useful BW.

Although the examples given which use CNOA's are for high-gain applications, it is easy to show that the deviation in amplitude and phase from the ideal is much lower than other existing realizations, even for closed-loop gains as low as unity. Also, another attractive feature of the proposed technique is that, in an integrated implementation, the chip area and the power consumption of a CNOA are much less than N times those of a single OA. This is because in a CNOA there is one output OA only that drives an external load and that may be required to have power handling capability.

In addition, it is worthwhile to mention that the method used to generate the CNOA's is actually a composite dependent source generation technique, i.e., it is applicable to any of the four types of dependent sources: voltage (and current) controlled voltage (or current) sources. Novel composite dependent sources with considerable performance improvements are expected to result when the procedure described is applied to the other dependent sources. Moreover, employing elements other than resistors for active compensation and using different types of OA's in the same CNOA (e.g., a high-accuracy OA for the input OA and a high-speed one for the output OA) are promising and challenging topics for further research and are presently under investigation.

It is to be noted that the CNOA's, when implemented using ideal OA's, represent ideal nullors independent of the absolute values of the compensating resistors. In certain configurations using CNOA's implemented using frequency-dependent OA's, the ratios of one or more of the compensating resistors to other resistors outside the CNOA appear in the high-order parasitic terms of T_a of the overall active realization. Although such terms are small, they can be made negligible by the proper choice of the impedance level of the appropriate compensating resistors. This is the approach taken here for simplicity while still yielding excellent results. Other researchers may find the impedance level as an added degree of freedom that may be used advantageously.

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