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OPERATIONAL DIFFERENTIATION USING SIMULATED INDUCTANCE WITH SINGLE-ENDED AMPLIFIERS

The passive *RL* form of differentiating circuit is used as a basis for the development of new forms of differentiator, in which the shunt inductance element *L* is simulated, and the error incurred by the series resistance *R* is compensated for, by a composite operational-amplifier network. An arrangement that employs only two single-ended operational amplifiers is derived.

It is well known in analogue-computer practice that difficulties are encountered when one seeks to mechanise differentiation; so that this operation is generally avoided. This problem has generated a variety of circuits, most of which are derivable from the passive *CR* differentiating circuit; but, to date, no widely accepted standard form has emerged. Some of the difficulties encountered are due to the occurrence of *C* in series with the signal path. If the alternative form is preferred, the series *C* will be replaced by *R* and the shunt *R* by *L*. It is now possible to simulate a grounded inductor quite accurately, using compact solid-state operational amplifiers. Accordingly, we have investigated operational differentiators based on this alternative approach, and have obtained a form that employs only two single-ended operational amplifiers with a minimum number of associated passive elements.

As a point of departure, consider the conventional operational integrator, with components C_1 and R_1, as shown in part of Fig. 1. If this circuit is bridged by another resistor

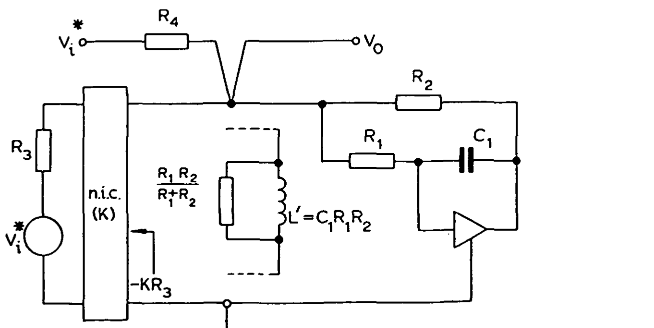

Fig. 1 *Operational form of RL differentiator with n.i.c. shunting simulated inductance.*
* indicates alternative inputs

R_2, the input admittance will have an equivalent inductance component $L' = C_1R_1R_2$ shunted by R_1 and by R_2. Such a stage is often employed, following a passive *CR* integrator, as an economical realisation of a second-order system having positive damping.[1] An obvious method of reducing or cancelling the loss components R_1, R_2 associated with L' is to shunt it by a negative resistance produced by a negative-impedance convertor (n.i.c.) loaded with a resistor R_3, with (for cancellation and a unity-ratio n.i.c., i.e. $K = 1$ in Fig. 1) $R_3 = R_1R_2/(R_1 + R_2)$. The voltage V_i (input) may be introduced across L' via a separate series resistor R_4 (this is the *R* of the basic *RL* differentiator), but this additional component may be avoided by inserting V_i in series with R_3 on the other side of the n.i.c., as shown in Fig. 1. By transforming V_i and R_3 (or R_4) into the equivalent current source $i = V_i/R_3$ (or V_i/R_4) and, in the former case, transferring the associated conductance $1/R_3$ (with reversal of sign) to the other side of the unity-ratio n.i.c., one can see that, with an ideal realisation, the result will be an ideal current source driving current through an ideal *L*, and exact differentiation will be obtained. This arrangement may be compared with the inductance-simulation circuit proposed by Ford and Girling,[2] which would require the additional series *R* if adapted for differentiation.

It will be appreciated that one can regard the amplifier and resistors R_1, R_2 in the bridge-*CR* integrator as an active realisation of a gyrator[3] that inverts the capacitor C_1 into the simulated inductance L', and hence that alternative forms of *RL* differentiator may be developed from other gyrator realisations.

There remains the realisation of the n.i.c.[4] If a differential-input operational amplifier may be used, one such unit connected as a current-inverting type of convertor in the well-known bridge configuration will suffice. But if single-ended amplifiers of the kind commonly employed in analogue computers must be used, two amplifiers in cascade, with an overall gain of -2, will be necessary, and the input resistor to the first stage will load the inductance L'. Moreover, the resistor R_3 under conversion will be bridged across the -2 amplifier chain, and therefore not in such a position that it may also be used for signal injection.

One way in which a separate 2-stage convertor may be avoided is shown in Fig. 2. A resistor R_1 is inserted in series with the

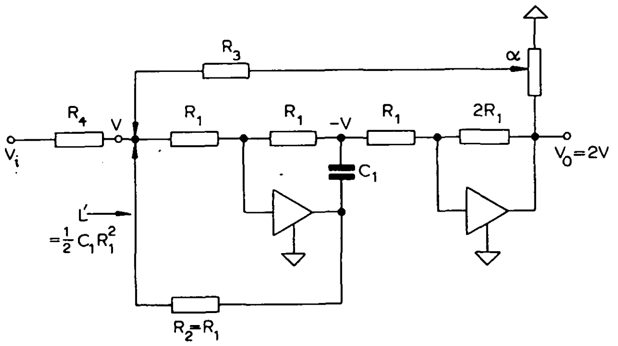

Fig. 2 *Operational differentiator with inductance simulation incorporated in loss-compensating feedback loop*

feedback capacitor C_1 of the integrator, on the summing-point side, to develop a voltage that is the negative of that across the simulated inductance L'. This voltage is multiplied by 2 and inverted by the second stage, which supplies a loss-compensating (positive-feedback) current to L', as well as providing a low-impedance output. It may be checked, most simply by T–π conversion of the T comprising the capacitor *C* and the two adjacent resistors R_1, that the effective capacitance in the integrator is now $C/2$ and $L'=\frac{1}{2}C_1R_1^2$. The entire 2-stage circuit may alternatively be regarded as an n.i.c. of the type described by Karplus,[4] with the addition of a quadrature feedback path in the first stage to produce a (lossy) positive shunt-inductance component of the input. Of course, with a capacitor in shunt with the simulated inductor, a bandpass filter is obtained whose selectivity depends on the amount of positive feedback, and, if this feedback is increased, oscillation will occur. The effective value of the resistor R_3 under negative conversion may be varied by connecting it to the convertor by a potentiometer of ratio α and, by straightforward analysis, one finds that its value is given by

$$R_3 = \frac{(2\alpha - 1)\,R_1\,R_4}{R_1 + 3R_4} \qquad \alpha > \tfrac{1}{2}$$

which reduces to $\frac{1}{4}R_1$ when $R_4 = R_1$ and $\alpha = 1$.

These circuits have been evaluated on the EAL type TR-20 analogue computer and the Tektronix type 3A8 operational-amplifier plug-in unit in the type 561A oscilloscope. Both are to be regarded as basic, in the sense that, for completely satisfactory differentiation, some degree of band limiting, preferably of adjustable width, to match the extent of the signal under differentiation, will be required. For most purposes, a 2-pole lowpass filter with variable cutoff frequency will be adequate, resulting in an overall transfer factor of the form $T(s) = as/(s^2 + bs + c)$.

Acknowledgment is due to the UK Science Research Council for support in the project from which this result arose.

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20th January 1967

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MAGNETICALLY TUNABLE MULTISECTION BANDPASS FILTERS IN FERRITE-LOADED EVANESCENT WAVEGUIDE

A description is given of a novel type of bandpass filter constructed in ferrite-loaded waveguide and using the evanescent-mode principle of operation. These filters may be multisection, and their operating band of frequencies may be tuned by a single-knob control of the d.c. magnetic field on the ferrite. Performance of a 3-section bandpass filter is given.

Recent publications[1,2] have shown that bandpass filters can be constructed in waveguide which is operating well below its cutoff frequency. The principle involved in these filters is that, although the attenuation (by reflection) of waveguide below cutoff is quite high, the network remains, to a close approximation, lossless. The present note, however, extends the above work to ferrite-loaded waveguide in which the cutoff frequency is controlled by the d.c. magnetic field on the ferrite.

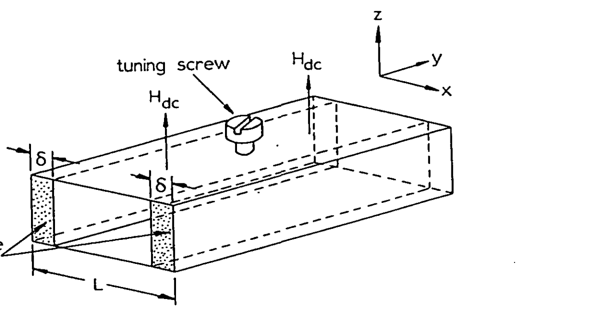

Fig. 1 *Single-section evanescent bandpass filter in ferrite-loaded waveguide*

As an example, consider the configuration shown in Fig. 1, in which two ferrite slabs (thickness δ) are positioned along the side walls of a section of rectangular waveguide. A transverse d.c. magnetic field is applied as shown. For the present analysis, it may be assumed that the r.f. fields vary as $\exp\{j(k_a x - \beta y)\}$ and $\exp(K_m x - j\beta y)$ in the empty and ferrite-filled regions, respectively. It will be noticed that the arrangement is symmetrical, in order to ensure reciprocity, which is a necessary condition for realising the filter.

Several authors[3,4] have analysed such an arrangement and have obtained, through the boundary-value problem, the characteristic equation for the propagating coefficient β; i.e.

$$\tan k_a(L - 2\delta) = p/q \quad \quad (1)$$

where $p = \dfrac{2K_m\mu_0}{\mu_e k_a}\cosh K_m\delta \sinh K_m\delta$

$$q = \left\{1 - \left(\frac{\beta\mu_0}{\mu_e\theta k_a}\right)^2\right\}\sinh^2 K_m\delta - \left(\frac{\mu_0 K_m}{\mu_e k_a}\right)^2\cosh^2 K_m\delta$$

$$\mu = \begin{pmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix} = \text{tensor permeability of the ferrite}$$

ϵ = permittivity of the ferrite

$$\mu_e = \frac{\mu^2 - k^2}{\mu} = \text{effective permeability of the ferrite}$$

$$\theta = \frac{\mu}{-jk}$$

$$K_m^2 = -\omega^2\epsilon\mu_e + \beta^2 \quad \quad (2)$$

$$k_a^2 = \omega^2\epsilon_0\mu_0 - \beta^2 \quad \quad (3)$$

It will be apparent from eqn. 1 that the propagation coefficient β occurs only as an even power, and thus the structure is reciprocal. To obtain the cutoff frequencies, the condition $\beta = 0$ put into eqn. 1 yields

$$\left(\frac{|\mu_e|\epsilon_0}{\mu_0\epsilon}\right)\tanh^2\{\omega_c(|\mu_e|\epsilon)^{1/2}\delta\} = \frac{1 + \cos\{\omega_c(\mu_0\epsilon_0)^{1/2}(L - 2\delta)\}}{1 - \cos\{\omega_c(\mu_0\epsilon_0)^{1/2}(L - 2\delta)\}} \quad (4)$$

ω_c = cutoff angular frequency

The d.c. magnetic field on the ferrite is such that μ_e is negative. The effect of this is that r.f. energy is excluded from the ferrite, and the cutoff frequency is increased above the value obtaining in empty waveguide. Thus, for any particular configuration and parameters, the cutoff frequency is obtained from eqn. 4.

The operating frequency of the device is well below its cutoff frequency; so that β is imaginary, and its value is obtained from eqn. 1.

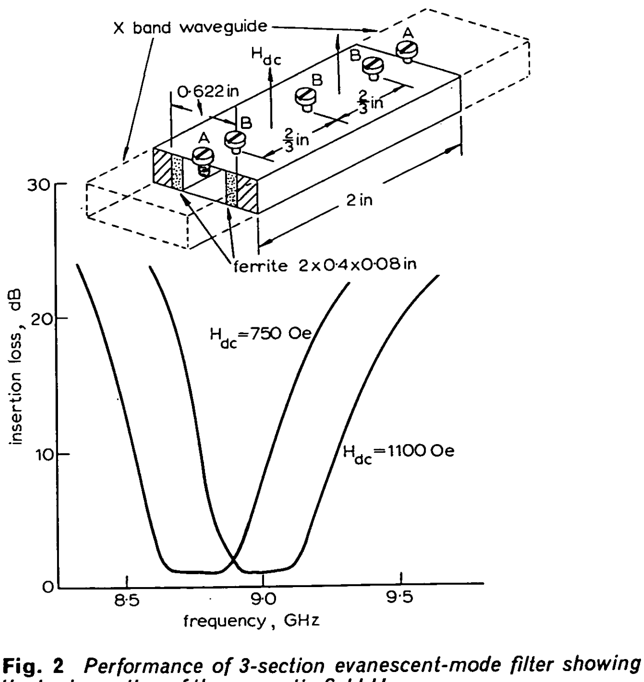

Fig. 2 *Performance of 3-section evanescent-mode filter showing the tuning action of the magnetic field H_{dc}*

Evanescence having been achieved, the remaining operation is to introduce into the section a capacitive tuning element such as a screw, as shown in Fig. 1, and to adjust its insertion